4-4 day 2 Optimization: Inscribing

Learning Objectives:

I can use derivatives to identify to optimize quantities in real world situations.

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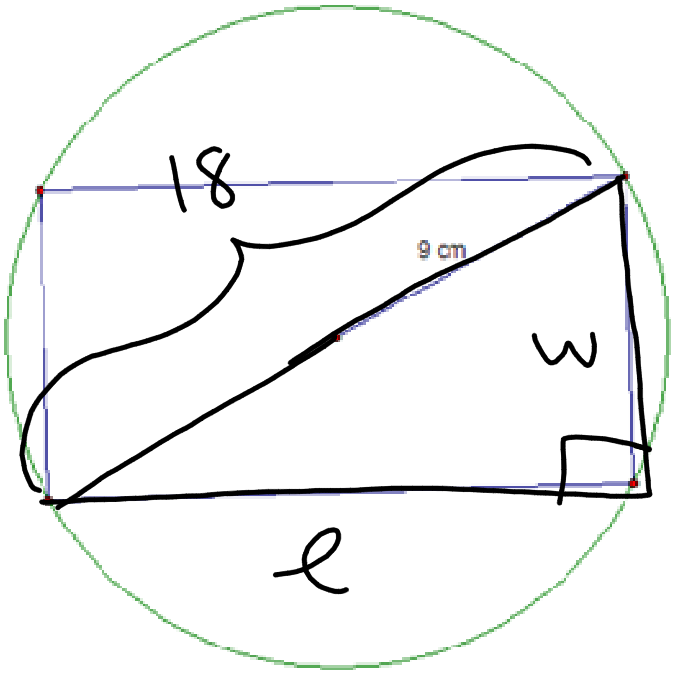
Ex1. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius 9 cm. What is the area of the largest rectangle?



objective:
Max Area ▭

$A = \ell \cdot w$

$A = w\sqrt{324 - w^2}$ (f, g)

$w^2 + \ell^2 = 18^2$

$w^2 + \ell^2 = 324$

$\ell^2 = 324 - w^2$

$\ell = \sqrt{324 - w^2}$

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$A = w\sqrt{324-w^2}$

$A = w(324-w^2)^{1/2}$

$f = w$ $\quad f' = 1$

$g = (324-w^2)^{1/2}$

$g' = \frac{1}{2}(324-w^2)^{-1/2} \cdot -2w$

$g' = \frac{-w}{\sqrt{324-w^2}}$

$A' = 1(324-w^2)^{1/2} + w \cdot \frac{-w}{\sqrt{324-w^2}}$

$A' = \sqrt{324-w^2} - \frac{w^2}{\sqrt{324-w^2}}$

$0 = \sqrt{324-w^2} - \frac{w^2}{\sqrt{324-w^2}}$

$\frac{w^2}{\sqrt{324-w^2}} = \frac{\sqrt{324-w^2}}{1}$

$w^2 = 324-w^2$

$\frac{2w^2}{2} = \frac{324}{2}$

$\sqrt{w^2} = \sqrt{162}$ $\quad w = \sqrt{162}$ cm

$l = \sqrt{324-w^2}$

$l = \sqrt{324-162}$

$l = \sqrt{162}$

$\sqrt{162}\text{cm} \times \sqrt{162}\text{cm}$

$A = 162\text{cm}^2$

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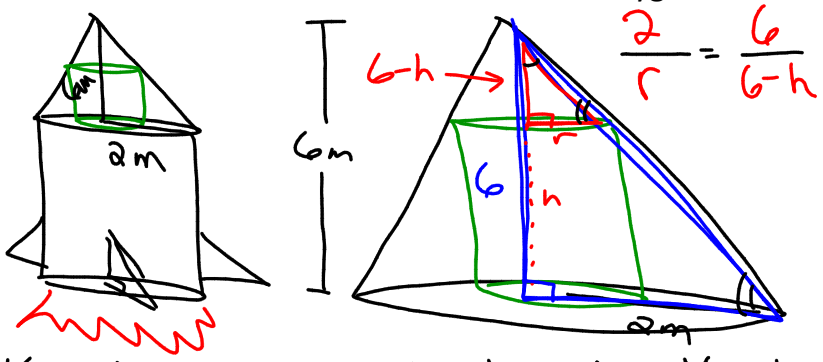
Ex2. The nose of rocket is a cone which is 6m high and has a radius of 2m. An oxygen tank is the shape of a cylinder and needs to be placed into the nose cone of the rocket. What dimensions should be used for the oxygen tank to maximize the volume of oxygen that the tank can hold? What is the volume of the oxygen tank?



$\frac{2}{r} = \frac{6}{6-h}$

$V(\text{cylinder}) = \pi r^2 h$

objective: Max V cyl

$12 - 2h = 6r$

$\frac{-2h}{-2} = \frac{6r-12}{-2}$

$h = -3r + 6$

$V = \pi r^2 h$

$V = \pi r^2 (-3r+6)$

$V = -3\pi r^3 + 6\pi r^2$

$V' = -9\pi r^2 + 12\pi r$

$0 = -9\pi r^2 + 12\pi r$

$= 3\pi r(-3r+4)$

$h = -3\left(\frac{4}{3}\right) + 6$

$h = -4 + 6$

$h = 2$

$3\pi r = 0$

$r = 0$

$-3r + 4 = 0$

$-3r = -4$

$r = \frac{4}{3}$ m

$h = 2$m

$V = \frac{32\pi}{9}$ m^3

$V = \pi(r^2)h$

$= \pi\left(\frac{4}{3}\right)^2 \cdot 2$

$= \pi \frac{16}{9} \cdot 2$

$= \frac{32\pi}{9}$

check $V'' = -18\pi r + 12\pi$

$= -18\pi\left(\frac{4}{3}\right) + 12\pi$

$-$ ccd max

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Homework

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